

What is claimed is:

1. A method of scaling up permeabilities associated with a fine-scale grid of cells representative of a porous medium to permeabilities associated with an unstructured coarse-scale grid of cells representative of the porous medium, comprising the steps of:
 - (a) generating an areally unstructured, Voronoi, computational grid using the coarse-scale grid as the genesis of the computational grid, the cells of the computational grid being smaller than the cells of the coarse-scale grid and each cell of said computational grid and said coarse-scale grid having a node;
 - (b) populating the computational grid with permeabilities associated with the fine-scale grid;
 - (c) developing flow equations for the computational grid, solving the flow equations, and computing inter-node fluxes and pressure gradients for the computational grid;
 - (d) using the fluxes and pressure gradients computed in step (c) to calculate inter-node average fluxes and average pressure gradients associated with the coarse-scale grid; and
 - (e) calculating upscaled permeabilities associated with the coarse-scale grid using the average fluxes and average pressure gradients calculated in step (d).
2. The method of claim 1 wherein inter-node connections of the computational grid cells are parallel to inter-node connections of the coarse-scale grid.
3. The method of claim 1 wherein cells of the computational grid are approximately the same size as the cells of the fine-scale grid.
4. The method of claim 1 wherein cells of the computational grid are smaller than the cells of the fine-scale grid.

5. The method of claim 1 wherein the permeabilities populated in step (b) are assigned to nodes of the computational grid, the permeability assigned to a given node of the computational grid corresponding to a predetermined permeability of a cell of the fine-scale grid that would contain the given node's location if the computational grid was superimposed on the fine-scale grid.
6. The method of claim 5 further comprises calculating the permeability for a given inter-node connection of the computational grid by harmonically averaging the permeabilities at the two nodes forming the given inter-node connection.
7. The method of claim 1 wherein the permeabilities populated in step (b) are assigned to midpoints of a given inter-node connection of the computational grid, the permeability assigned to the given inter-node connection corresponding to a predetermined permeability of a cell of the fine-scale grid that would contain the mid-point of such inter-node connection if the computational grid was superimposed on the fine-scale grid.
8. The method of claim 1 wherein the calculation of the inter-node average fluxes and average pressure gradients in step (d) is determined by using only the fluxes and pressure gradients computed for the inter-node connections of the computational grid that fall within a predetermined sub-domain of the computational node.
9. The method of claim 1 wherein the calculation of the average fluxes and average pressure gradients in step (d) for a given inter-node connection of the coarse-scale grid is determined using only fluxes and pressure gradients computed in step (c) for the inter-node connections of the computational grid that are parallel to the given inter-node connection.

10. The method of claim 1 wherein the permeabilities calculated in step (e) are determined for a given node of the coarse-scale grid.
- 5 11. The method of claim 1 wherein the permeabilities calculated in step (e) are determined for a given inter-node connection of the coarse-scale grid by calculating the ratio of the average flux to the average pressure gradient computed in step (d) for the given inter-node connection.
- 10 12. The method of claim 1 wherein cells of the fine-scale grid are structured.
13. The method of claim 1 wherein the coarse-scale grid is a PEBI grid.
14. The method of claim 1 wherein both the coarse-scale grid and the computational grid are PEBI grids.
- 15 15. The method of claim 1 wherein inter-node connections of the coarse grid forms Delaunay triangles and the computational grid generated in step (a) contains similar, smaller Delaunay triangles equal in number to n^2 times the number of Delaunay triangles of the large-scale grid, where n is a predetermined integer refinement ratio used to generate the computational grid.
- 20 16. The method of claim 1 wherein all cells are three-dimensional.
17. The method of claim 16 wherein the coarse-scale grid and the computational grid are both unstructured areally and structured vertically.
- 25 18. The method of claim 1 further comprises determining inter-node connection transmissibilities of the coarse-scale grid using permeabilities calculated in step (e).
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19. The method of claim 1 wherein the flow equations of step (c) are single-phase and steady-state.
20. A method for estimating permeability of each cell of a first grid having a
multiplicity of cells of a subterranean geologic domain using a predetermined
permeability for each cell of a second grid representative of the domain, said
second grid containing a larger number of cells than the first grid, the method
comprising:
- (a) constructing an unstructured, third grid representative of the domain
comprising approximately the same or greater number of cells than the
second grid, each cell of the first, second, and third grids having a node
and each link between two nodes of adjacent cells being an inter-node
connection, substantially all of the inter-node connections of the third
grid being parallel to the inter-node connections of the first grid;
 - (b) for each node of the third grid, assigning a permeability corresponding to
the permeability of a cell of the second grid that contains the node
location of the third grid;
 - (c) developing a single-phase, steady-state pressure equation for each cell of
the third grid system;
 - (d) solving the pressure equations and computing fluxes and pressure
gradients for all inter-node connections of the third grid;
 - (e) computing an estimated permeability for a given connection of the first
grid using inter-node connections of the third grid; and
 - (f) repeating step (e) for all connections of the first grid.
21. The method of claim 20 further comprises computing the permeability in
step (e) by the additional steps of determining average fluxes and average
pressure gradients over sub-domains associated with a given grid inter-node
connection of the first grid and calculating a ratio of the average flux to the
average pressure gradient, thereby obtaining the permeability for the given
inter-node connection of the first grid.

22. The method of claim 20 wherein the permeability computation of step (e) uses only inter-node connections of the third grid that are parallel to inter-node connections of the first grid.

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23. The method of claim 20 wherein all grid cells are three-dimensional.

24. The method of claim 23 wherein the second grid and the third grid are each unstructured areally and structured vertically.

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25. A method for estimating permeabilities associated with cells of a large-scale grid representative of fluid flow in a porous medium using predetermined permeabilities associated with cells of a small-scale grid also representative of fluid flow in the porous medium, each cell of the large-scale grid having a node and each node being linked to adjacent nodes to form inter-node connections and such connections forming Delaunay triangles, comprising the steps of:

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(a) constructing a computational grid by dividing each Delaunay triangle of the large-scale grid into a multiplicity of similar, smaller Delaunay triangles, the sides of such smaller Delaunay triangles being inter-node connections of the computational grid and the inter-node connections of the large-scale grid and the computational grid being aligned with each other;

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(b) assigning permeabilities to the computational grid corresponding to the predetermined permeabilities of the small-scale grid;

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(c) developing a single-phase, steady-state pressure equation for each cell of the computational grid, solving the pressure equations, and computing fluxes and pressure gradients for all inter-node connections of the computational grid;

- 5 (d) using the fluxes and pressure gradients computed in step (c) to calculate an average flux and an average pressure gradient for each inter-node connection of the large-scale grid; and
- (e) calculating a permeability associated with a given inter-node connection of the large-scale grid using the average flux and average pressure gradient calculated in step (d).

- 10 26. The method of claim 25 wherein the number of nodes of the computational grid are approximately the same as the number of cells of the small-scale grid.
27. The method of claim 25 wherein the number of nodes of the computational grid are smaller than the number of cells of the small-scale grid.
- 15 28. The method of claim 25 wherein the number of similar, smaller Delaunay triangles is n^2 times the number of Delaunay triangles of the large-scale grid, where n is a predetermined integer refinement ratio.